3.1 Lines and Angles

What you should learn

GOAL 1 Identify relationships between lines.

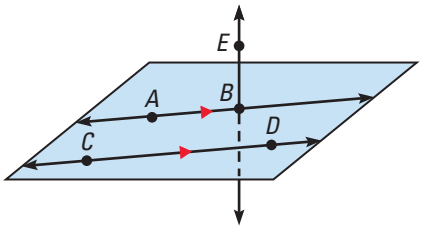
GOAL 2 Identify angles formed by transversals.

Why you should learn it

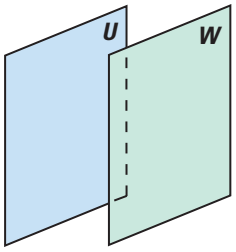
▼ To describe and understand **real-life** objects, such as the escalator in **Exs. 32–36**.

GOAL 1 RELATIONSHIPS BETWEEN LINES

Two lines are **parallel lines** if they are coplanar and do not intersect. Lines that do not intersect and are not coplanar are called **skew lines**. Similarly, two planes that do not intersect are called **parallel planes**.



$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines.
$\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$ are skew lines.



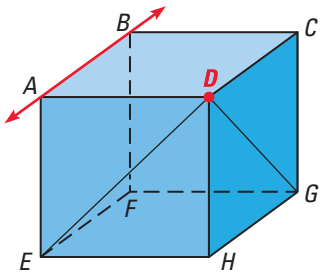
Planes *U* and *W* are parallel planes.

To write "$\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$," you write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Triangles like those on $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are used on diagrams to indicate that lines are parallel.

Segments and rays are parallel if they lie on parallel lines. For example, $\overline{AB} \parallel \overline{CD}$.

EXAMPLE 1 *Identifying Relationships in Space*

Think of each segment in the diagram as part of a line. Which of the lines appear to fit the description?



a. parallel to $\overleftrightarrow{AB}$ and contains D

b. perpendicular to $\overleftrightarrow{AB}$ and contains D

c. skew to $\overleftrightarrow{AB}$ and contains D

d. Name the plane(s) that contain D and appear to be parallel to plane ABE.

SOLUTION

a. $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$, and $\overleftrightarrow{EF}$ are all parallel to $\overleftrightarrow{AB}$, but only $\overleftrightarrow{CD}$ passes through D and is parallel to $\overleftrightarrow{AB}$.

b. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{AE}$, and $\overleftrightarrow{BF}$ are all perpendicular to $\overleftrightarrow{AB}$, but only $\overleftrightarrow{AD}$ passes through D and is perpendicular to $\overleftrightarrow{AB}$.

c. $\overleftrightarrow{DG}$, $\overleftrightarrow{DH}$, and $\overleftrightarrow{DE}$ all pass through D and are skew to $\overleftrightarrow{AB}$.

d. Only plane DCH contains D and is parallel to plane ABE.

STUDENT HELP

Look Back
For help identifying perpendicular lines, see p. 79.

Notice in Example 1 that, although there are many lines through D that are skew to $\overleftrightarrow{AB}$, there is only one line through D that is parallel to $\overleftrightarrow{AB}$ and there is only one line through D that is perpendicular to $\overleftrightarrow{AB}$.

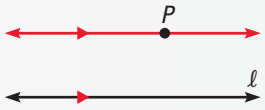
STUDENT HELP

APPLICATION LINK Visit our Web site www.mcdougallittell.com for more information about the parallel postulate.

PARALLEL AND PERPENDICULAR POSTULATES

POSTULATE 13 ***Parallel Postulate***

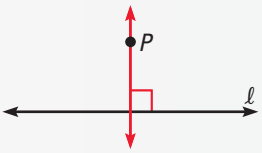
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through *P* parallel to ℓ.

POSTULATE 14 ***Perpendicular Postulate***

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through *P* perpendicular to ℓ.

You can use a compass and a straightedge to construct the line that passes through a given point and is perpendicular to a given line. In Lesson 6.6, you will learn why this construction works.

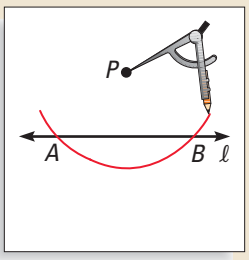
You will learn how to construct a parallel line in Lesson 3.5.

ACTIVITY

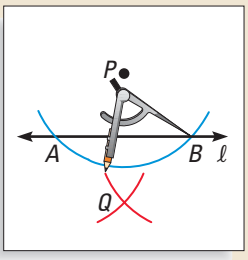
Construction

A Perpendicular to a Line

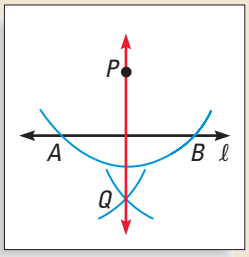
Use the following steps to construct a line that passes through a given point P and is perpendicular to a given line ℓ.



1 Place the compass point at P and draw an arc that intersects line ℓ twice. Label the intersections A and B.



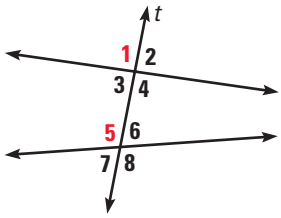
2 Draw an arc with center A. Using the same radius, draw an arc with center B. Label the intersection of the arcs Q.



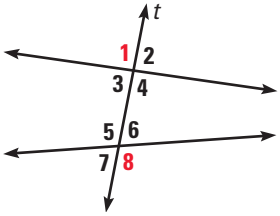
3 Use a straightedge to draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp \ell$.

GOAL 2 IDENTIFYING ANGLES FORMED BY TRANSVERSALS

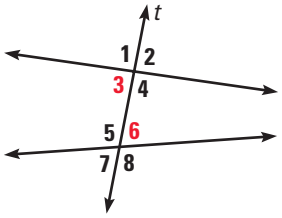
A **transversal** is a line that intersects two or more coplanar lines at different points. For instance, in the diagrams below, line t is a transversal. The angles formed by two lines and a transversal are given special names.



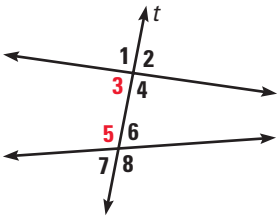
Two angles are **corresponding angles** if they occupy corresponding positions. For example, angles **1** and **5** are corresponding angles.



Two angles are **alternate exterior angles** if they lie outside the two lines on opposite sides of the transversal. Angles **1** and **8** are alternate exterior angles.



Two angles are **alternate interior angles** if they lie between the two lines on opposite sides of the transversal. Angles **3** and **6** are alternate interior angles.



Two angles are **consecutive interior angles** if they lie between the two lines on the same side of the transversal. Angles **3** and **5** are consecutive interior angles.

Consecutive interior angles are sometimes called **same side interior angles**.

EXAMPLE 2 *Identifying Angle Relationships*

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

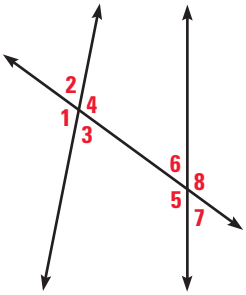
List all pairs of angles that fit the description.

a. corresponding

b. alternate exterior

c. alternate interior

d. consecutive interior



SOLUTION

a. $\angle 1$ and $\angle 5$
$\angle 2$ and $\angle 6$
$\angle 3$ and $\angle 7$
$\angle 4$ and $\angle 8$

b. $\angle 1$ and $\angle 8$
$\angle 2$ and $\angle 7$

c. $\angle 3$ and $\angle 6$
$\angle 4$ and $\angle 5$

d. $\angle 3$ and $\angle 5$
$\angle 4$ and $\angle 6$

GUIDED PRACTICE

Vocabulary Check ✓

1. Draw two lines and a transversal. Identify a pair of alternate interior angles.

Concept Check ✓

2. How are skew lines and parallel lines alike? How are they different?

Skill Check ✓

Match the photo with the corresponding description of the chopsticks.

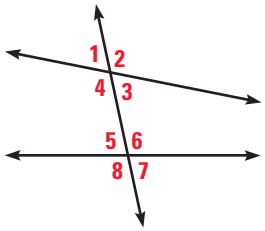
A. skew **B.** parallel **C.** intersecting

3. 

4. 

5. 

In Exercises 6–9, use the diagram at the right.



6. Name a pair of corresponding angles.
7. Name a pair of alternate interior angles.
8. Name a pair of alternate exterior angles.
9. Name a pair of consecutive interior angles.

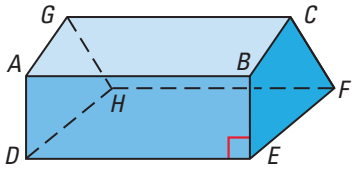
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 807.

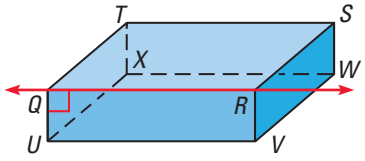
LINE RELATIONSHIPS **Think of each segment in the diagram as part of a line. Fill in the blank with *parallel*, *skew*, or *perpendicular*.**

10. $\overleftrightarrow{DE}$, $\overleftrightarrow{AB}$, and $\overleftrightarrow{GC}$ are ___?___.
11. $\overleftrightarrow{DE}$ and $\overleftrightarrow{BE}$ are ___?___.
12. $\overleftrightarrow{BE}$ and $\overleftrightarrow{GC}$ are ___?___.
13. Plane GAD and plane CBE are ___?___.



IDENTIFYING RELATIONSHIPS **Think of each segment in the diagram as part of a line. There may be more than one right answer.**

14. Name a line parallel to $\overleftrightarrow{QR}$.
15. Name a line perpendicular to $\overleftrightarrow{QR}$.
16. Name a line skew to $\overleftrightarrow{QR}$.
17. Name a plane parallel to plane QRS.



STUDENT HELP

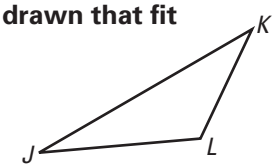
HOMEWORK HELP

Example 1: Exs. 10–20, 27–36

Example 2: Exs. 21–26

APPLYING POSTULATES **How many lines can be drawn that fit the description?**

18. through L parallel to $\overleftrightarrow{JK}$
19. through L perpendicular to $\overleftrightarrow{JK}$



FOCUS ON PEOPLE

PHILIPPE PETIT walked more than 2000 feet up an inclined cable to the Eiffel Tower. The photo above is from a performance in New York City.

20. TIGHTROPE WALKING Philippe Petit sometimes uses a long pole to help him balance on the tightrope. Are the rope and the pole at the left *intersecting*, *perpendicular*, *parallel*, or *skew*?

ANGLE RELATIONSHIPS Complete the statement with *corresponding, alternate interior, alternate exterior,* or *consecutive interior*.

21. $\angle 8$ and $\angle 12$ are ___?___ angles.

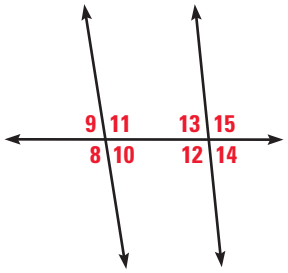
22. $\angle 9$ and $\angle 14$ are ___?___ angles.

23. $\angle 10$ and $\angle 12$ are ___?___ angles.

24. $\angle 11$ and $\angle 12$ are ___?___ angles.

25. $\angle 8$ and $\angle 15$ are ___?___ angles.

26. $\angle 10$ and $\angle 14$ are ___?___ angles.



ROMAN NUMERALS Write the Roman numeral that consists of the indicated segments. Then write the base ten value of the Roman numeral. For example, the base ten value of XII is 10 + 1 + 1 = 12.

Roman numeral	I	V	X	L	M
Base ten value	1	5	10	50	1000

27. Three parallel segments

28. Two non-congruent perpendicular segments

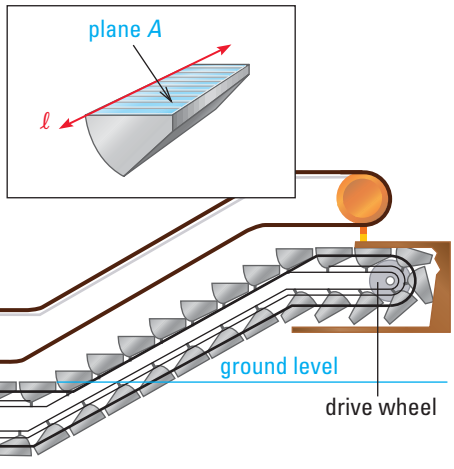
29. Two congruent segments that intersect to form only one angle

30. Two intersecting segments that form vertical angles

31. Four segments, two of which are parallel

ESCALATORS In Exercises 32–36, use the following information.

The steps of an escalator are connected to a chain that runs around a drive wheel, which moves continuously. When a step on an up-escalator reaches the top, it flips over and goes back down to the bottom. Each step is shaped like a wedge, as shown at the right. On each step, let plane A be the plane you stand on.



32. As each step moves around the escalator, is plane A always parallel to ground level?

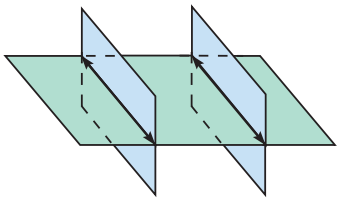
33. When a person is standing on plane A, is it parallel to ground level?

34. Is line ℓ on any step always parallel to ℓ on any other step?

35. Is plane A on any step always parallel to plane A on any other step?

36. As each step moves around the escalator, how many positions are there at which plane A is perpendicular to ground level?

37. **LOGICAL REASONING** If two parallel planes are cut by a third plane, explain why the lines of intersection are parallel.



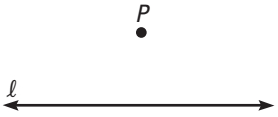
38. *Writing* What does "two lines intersect" mean?

39. **CONSTRUCTION** Draw a horizontal line ℓ and a point P above ℓ. Construct a line through P perpendicular to ℓ.

40. **CONSTRUCTION** Draw a diagonal line m and a point Q below m. Construct a line through Q perpendicular to m.

Test Preparation

41. **MULTIPLE CHOICE** In the diagram at the right, how many lines can be drawn through point P that are perpendicular to line ℓ?



(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3

42. **MULTIPLE CHOICE** If two lines intersect, then they must be ___?___.

(A) perpendicular (B) parallel (C) coplanar (D) skew (E) None of these

★ Challenge

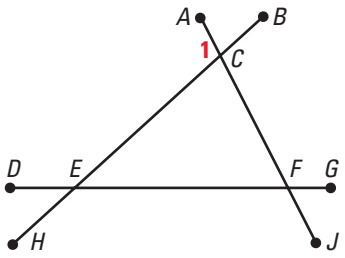
ANGLE RELATIONSHIPS **Complete each statement. List all possible correct answers.**

43. $\angle 1$ and __?__ are corresponding angles.

44. $\angle 1$ and __?__ are consecutive interior angles.

45. $\angle 1$ and __?__ are alternate interior angles.

46. $\angle 1$ and __?__ are alternate exterior angles.



EXTRA CHALLENGE
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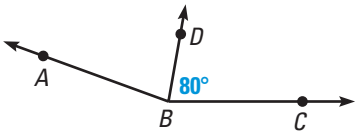
MIXED REVIEW

47. **ANGLE BISECTOR** The ray $\overrightarrow{BD}$ bisects $\angle ABC$, as shown at the right. Find $m\angle ABD$ and $m\angle ABC$. (Review 1.5 for 3.2)



COMPLEMENTS AND SUPPLEMENTS **Find the measures of a complement and a supplement of the angle.** (Review 1.6 for 3.2)

48. 71° **49.** 13° **50.** 56°

51. 88° **52.** 27° **53.** 68°

54. 1° **55.** 60° **56.** 45°

WRITING REASONS **Solve the equation and state a reason for each step.** (Review 2.4 for 3.2)

57. $x + 13 = 23$ **58.** $x - 8 = 17$ **59.** $4x + 11 = 31$

60. $2x + 9 = 4x - 29$ **61.** $2(x - 1) + 3 = 17$ **62.** $5x + 7(x - 10) = -94$